

White Dwarfs in Globular Clusters: *HST* Observations of M4[†]

Harvey B. Richer¹, Gregory G. Fahlman¹, Rodrigo A. Ibata¹, Carlton Pryor², Roger A. Bell³, Michael Bolte⁴, Howard E. Bond⁵, William E. Harris⁶, James E. Hesser⁷, Steve Holland¹, Nicholas Ivanans¹, Georgi Mandushev¹, Peter B. Stetson⁷ & Matt A. Wood⁸

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¹Department of Physics & Astronomy, University of British Columbia, Vancouver, B.C., V6T 1Z4. E-mail surname@astro.ubc.ca

²Rutgers University, Department of Physics and Astronomy, PO Box 849, Piscataway, NJ 08855-0849. E-mail pryor@physics.rutgers.edu

³University of Maryland, Department of Astronomy, College Park, MD 20742-2421. E-mail rabell@astro.umd.edu

⁴University of California, Lick Observatory, Santa Cruz, CA 95064. E-mail bolte@ucolick.org

⁵Space Telescope Science Institute, 3700 San Martin Drive, Baltimore, MD 21218. E-mail bond@stsci.edu

⁶McMaster University, Department of Physics and Astronomy, Hamilton, ON, Canada L8S 4M1. E-mail harris@physun.physics.mcmaster.ca

⁷Dominion Astrophysical Observatory, Herzberg Institute of Astrophysics, National Research Council, 5071 W. Saanich Road, RR5, Victoria, B.C., Canada V8X 4M6. E-mail firstname.lastname@hia.nrc.ca

⁸Florida Institute of Technology, Dept. Physics & Space Sciences, 150 W. University Blvd, Melbourne, FL 32901-6988. Email wood@kepler.pss.fit.edu

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ABSTRACT

Using WFPC2 on the *Hubble Space Telescope*, we have isolated a sample of 258 white dwarfs (WDs) in the Galactic globular cluster M4. Fields at three radial distances from the cluster center were observed and sizeable WD populations were found in all three. The location of these WDs in the color-magnitude diagram, their mean mass of $0.51(\pm 0.03)M_{\odot}$, and their luminosity function confirm basic tenets of stellar evolution theory and support the results from current WD cooling theory. The WDs are used to extend the cluster main-sequence mass function upward to stars that have already completed their nuclear evolution. The WD/red dwarf binary frequency in M4 is investigated and found to be at most a few percent of all the main-sequence stars. The most ancient WDs found are ~ 9 Gyr old, a level which is set solely by the photometric limits of our data. Even though this is less than the age of M4, we discuss how these cooling WDs can eventually be used to check the turnoff ages of globular clusters and hence constrain the age of the Universe.

Subject headings: clusters: globular, clusters: individual M4, stars: white dwarfs

1. Introduction

Stellar evolution theory predicts that all single, low-mass stars end their lives as white dwarfs (WDs). This fundamental prediction has, however, never actually been observationally confirmed. Attempts in this direction were made by Weidemann (1989) who compared the planetary nebulae birth rate with that of WDs, but later showed (Weidemann 1990) that not all WDs go through the planetary nebula evolutionary channel. White dwarfs in open clusters have been observed to determine ages, the upper mass limit of WD progenitors and relative birthrate statistics (*e.g.* Luyten & Herbig 1960; Woolf 1974; Hartwick & Hesser 1978; Romanishin & Angel 1980; Anthony-Twarog 1981, 1982; Koester & Reimers 1982, 1993, 1996; von Hippel *et al.* 1996), but the population of WDs in any open cluster is always small. What is required are observations of a rich stellar cluster that penetrate faint enough so that the WD members can be found in sufficient numbers. The number of WDs detected can then be compared with a theoretical prediction based on the number of bright stars in some well-defined evolutionary phase with its associated timescale, and the WD cooling time to the limit of detection. Clearly, globular star clusters are an excellent testing ground for such an approach. No convincing WD sample was ever isolated in a globular cluster from ground-based work despite several attempts (Richer 1978; Chan & Richer 1986; Ortolani & Rosino 1987; Richer & Fahlman 1988) as the distances to the clusters makes the WDs faint and the crowding of the stellar images in their dense inner regions severe.

With the repair of *HST*, crowding no longer remains as a barrier and there are three recent reports of significant populations of WDs in the nearest clusters (Richer *et al.* 1995, Cool *et al.* 1996, Renzini *et al.* 1996). The first of these papers announced a well-populated WD sequence in M4 and derived a preliminary luminosity function for them. Cool *et al.* discovered a modest population of a few dozen WDs in NGC 6397 while Renzini *et al.* used

the WD cooling sequence in NGC 6752 to derive an accurate distance to the cluster. In the present paper we discuss the largest sample of WDs yet discovered in a stellar cluster, that in the Galactic globular cluster M4. We find that the M4 WDs possess an average mass ($0.51M_{\odot}$) very close to that expected from stellar evolution theory (§5), and that they appear in numbers consistent with the hypothesis that *all* evolving stars in the cluster for the past 4 Gyr have terminated their lives as WDs (§6).

The oldest WDs which we observe in M4 have cooling ages of ~ 9 Gyr. This detection limit is currently governed only by the length of the *HST* exposures. In §9 we discuss whether globular cluster WDs can be detected to cosmologically interesting ages and thus whether they can be used to check cluster turnoff ages and hence constrain the age of the Universe.

2. Observations and Data Reduction

The observations discussed in the following sections are imaging data obtained with *HST* using WFPC2 in cycle 4 (GO-5461). Three fields were imaged: one whose PC field was within one core radius (r_c) of the cluster center, which we take as $50''$ (Djorgovski 1993); a second whose PC field was at about $1.7r_c$; and a third whose PC field was near $6r_c$. In the inner two fields the cluster was imaged through three filters: F336W (U), F555W (V) and F814W (I). For the outer field we only exposed through the two reddest filters. The aim of the program was to use the long color baseline of F336W - F814W ($U - I$) in those regions where we expected the largest number of WDs so that we could best define the WD cooling sequence, and to reach as faint as possible for the cooler WDs and the redder main sequence stars in the outermost uncrowded field. Total exposure times in the two inner fields were 11,800, 15,000 and 5,500 seconds in F336W, F555W and F814W, respectively. In the $6r_c$ field the exposures were 31,500 seconds in F555W and 5,500 seconds in F814W.

The locations of the measured stars in these fields are illustrated in Figure 1 where annuli boundaries at 0.5 , 1.5 , 2.5 , 4 and $8r_c$ are displayed centered on the cluster together with all of the stars found in each of them. In the discussion which follows we will be referring to the stars by annulus with annulus 1 lying between 0.5 and $1.5r_c$, annulus 2 between 1.5 and $2.5r_c$, annulus 3 between 2.5 and $4.0r_c$ and with annulus 4’s PC field centered near $6.0r_c$. At the time when the data were taken, it was not as yet clear whether it would be possible to reduce the badly undersampled data from the WF cameras. Experience has now taught us that the WF data are almost as reliable as those of the PC, but at the time of setting up the program, we made sure that there were no very bright stars on the PC frames and we let the WF cameras fall as they may without trying to control the spacecraft’s roll angle. For this reason there is quite a bit of overlap in annulus 2, i.e. a large fraction of these stars were measured in two separate pointings of the telescope.

The raw *HST* data frames had the standard pipeline processing performed on them; this includes bias subtraction, correction for dark current, and flat-fielding. In addition to this, we flagged hot pixels and did not use them in the reductions, vignetted pixels were blanked out, the charge transfer efficiency correction was made, the decrease in F336W transmission as a function of time from the last decontamination was accounted for, and we corrected for nonuniform illumination of the chips. The photometry was carried out using ALLFRAME (Stetson 1987, 1994) with a quadratically-varying point-spread function (PSF). Complete details concerning these reductions can be found in Ibata *et al.* (1997).

This technique of PSF-fitting in deriving magnitudes and colors for stars requires that we are able to relate a PSF magnitude, which is only sampled around the intensity peak, to one derived from a large aperture which envelopes the entire light from the star. Instead of attempting to derive the aperture corrections to the PSF magnitudes directly from the data (which are often badly corrupted by cosmic rays), we produced a number of artificial

frames (containing appropriate sky noise) which contained realizations of the PSF free of cosmic rays and measured the corrections on these artificial frames.

To permit comparison with theoretical results and with ground-based photometry, all of the data were transformed to U , V and I as defined by the Landolt standards (1983, 1992a,b) using the transformations discussed in Holtzman *et al.* (1995). In addition to this transformation, we included the +0.05 mag correction which Kelson *et al.* (1996) have shown is required for *HST* images which have in excess of $160e^-$ in each sky pixel. For sky counts less than $160e^-$ a linear ramp between no correction and +0.05 mags was used. As a check on these procedures, in Figure 2 we compare photometry of individual stars from annulus 4 obtained with the Las Campanas Observatory 2.5 m telescope (calibrated with Landolt standards) with that from *HST* transformed to ground-based V and I , as described above. These diagrams show quite satisfactory agreement between the two data sets. There is some evidence of a small systematic difference between the photometric zeropoints of the two systems amounting to about 0.03 mags in V and 0.01 mags in I . Since it is not clear whether the ground or the space data might contain this small error, we have not forced the two sets to agree but have adopted the calibration for the *HST* data as is.

A further check on the calibration of the photometry involves a comparison between the U , $(U - I)$ CMD from the ground and from space. The ground U photometry was obtained from CTIO (Richer & Fahlman 1984). There are no stars in common for this check (our ground-based U photometry was also obtained in annulus 4, for which no F336W frames were secured), so all we can compare are the resultant CMDs (Figure 3). The agreement between the locations of the major sequences of the two CMDs provides confidence in the calibration of the *HST* data that we have adopted.

In Table 1 we list the coordinates and photometry of the 258 WDs found in the three fields. The objects appearing in this table all possess χ -parameters (which is a measure

of the goodness of fit between the stellar profile and the point spread function) that are ≤ 2.0 (Stetson 1987) and photometric errors in their V and I colors that are no larger than 0.5 mags. In a few cases the U errors are very much larger than this, which reflects the faintness in this color of the cooler WDs. The column headings are as follows: column 1, original field number F (0 = core field, 1 = $1r_c$ field, 6 = $6r_c$ field); column 2, C the chip on which the WD is located (1 = PC, 2, 3 and 4 the WF chips as numbered in the *HST* Handbook); columns 3 and 4, the X,Y pixel coordinates of the star on the chip; columns 5 and 6, right ascension and declination (equinox 2000), both measured in degrees and derived from the pointing of the spacecraft and the pixel coordinates using the software in STSDAS; columns 7 and 8, the U mag. and error; columns 9 and 10, the V mag. and error; and columns 11 and 12, the I mag. and error. The error estimates listed are the rms frame-to-frame variations in the measured magnitudes.

3. The Distance and Reddening to M4

Both the distance and reddening to M4 are somewhat controversial. This is likely due in part to the fact that the cluster is located in the direction of the Scorpius-Ophiuchus dark cloud, so that the reddening is quite high. A large reddening also introduces the possibility of differential extinction across the face of the cluster (Cudworth & Rees 1990a). Several recent papers (Liu & Janes 1990, Dixon & Longmore 1993, Vrba *et al.* 1993, Peterson *et al.* 1995) have further suggested that the ratio of total-to-selective absorption in the V -band in the direction of M4, R_V , is larger than normal, in the range of 3.8. We adopt this value for the discussion which follows below.

The cluster distance and reddening are derived simultaneously here by fitting a set of subdwarfs to the unevolved part of the M4 main sequence. The subdwarfs are the same sample used by Mandushev *et al.* (1996) in their study of M55 and the M4 color-magnitude

diagram (CMD) employed is the ground-based data used above to check the calibration of the *HST* photometry. We used these data instead of the *HST* results as the ground-based data contain brighter stars which better match the subdwarf sample. The stars in the upper part of the unevolved main sequence are largely saturated on the *HST* frames.

The theoretical models of Bertelli *et al.* (1994) were used differentially to adjust the subdwarf colors to account for their metallicity difference with respect to M4’s, which is taken to be $[Fe/H] = -1.3$ (Djorgovski 1993). The best fit between this empirical population II main sequence and the fiducial sequence of the cluster is shown in Figure 4. Contours of constant χ^2 , the goodness of fit between the M4 fiducial sequence and the subdwarfs for different reddening and distance moduli are illustrated in Figure 5. It is clear from the diagram that this main-sequence fitting technique is capable of constraining the reddening reasonably well, but does a less accurate job of determining the best estimate of the apparent distance modulus.

The parameter values that minimize χ^2 are $(m - M)_V = 12.51(\pm 0.09)$ and $E(V - I) = 0.51(\pm 0.02)$. We will use these values for the rest of this paper. With $R_V = 3.8$, we can then derive that $E(B - V) = 0.35$, $A_V = 1.32$ and, thus, that the distance to M4 is 1.73 kpc. The expressions developed in Cardelli *et al.* (1989) were used to determine $E(B - V)$ from $E(V - I)$ with our non-standard value of R_V . The present value for $E(B - V)$ is in excellent agreement with direct measurements of this quantity in M4 (e.g., Richer & Fahlman 1984; Dixon & Longmore 1993). Moreover, the rather small cluster distance is in almost perfect accord with the astrometrically determined distance of 1.72 ± 0.14 kpc (Peterson *et al.* 1995) and the 1.73 kpc derived from the Baade-Wesselink analysis of M4 RR Lyrae stars (Liu & Janes 1990).

4. The White Dwarf Cooling Sequence

Since all single stars currently completing their nuclear evolution in a globular cluster end up as WDs, and since this statement has been true for many billions of years now, we expect a substantial population of WDs to be present in M4. Thus our first aim is to demonstrate whether WDs are indeed present and whether they are located in the expected regions of the CMD. Section 6 of this paper will address the question of whether the WDs are present in the expected numbers.

The left-hand panel of Figure 6 displays the M_U , $(U - I)_O$ CMD for all stars possessing U photometry (annuli 1-3) with errors in each color that are less than 0.25 mag and $\chi < 1.5$ (Stetson 1987). The appearance of this CMD differs somewhat from our earlier version (Richer *et al.* 1995) with these same data, as we have reprocessed the images with a smaller fitting radius which is less capable of measuring saturated stars, but does a better job with the faintest objects. The long color baseline was chosen to make the WD cooling sequence as distinct as possible from that of the main sequence. Two well-defined sequences are present in this diagram, the main sequence of the cluster and a roughly parallel sequence containing 109 stars about five magnitudes bluer in $(U - I)$ that begins at U magnitude near 22 ($M_U = 9$) and continues to the limit of the data at $M_U = 12.5$ ($U = 25.5$).

The scatter of stars below the main sequence is contributed by the Galactic bulge/spheroid as M4 is roughly in the Galactic center direction. Additional discussion of this component of the CMD can be found in Fahlman *et al.* (1996a, 1996b, 1997). The locus of the bluest stars is certainly what a WD cooling sequence is expected to look like. We confirm that suspicion by replotting the data and overlaying a theoretical cooling curve for $0.5M_\odot$ hydrogen-rich (DA) WDs (Figure 6, right-hand panel). The theoretical locus was derived from the evolutionary models of Wood (1995) and the DA atmospheres of Bergeron *et al.* (1995). Wood’s models provide the WD mass and radius at a given T_{eff} or age. From

these numbers, the gravity of the star was calculated. We then interpolated in the table of atmospheric colors of Bergeron *et al.* to obtain colors at the appropriate gravity and T_{eff} . The agreement between the theoretical and observed loci makes it seem incontrovertible that the sequence of blue objects is indeed the cluster WD cooling sequence.

We note in passing the hot (27,000 K) WD at $M_U = 8.5$; according to the evolutionary models of Wood (1995), a WD at this temperature is only 13 million years old. Neutrino losses, while somewhat less than the photon luminosity of the star, are still an important energy loss mechanism for this WD. This star is in the region of the WD cooling sequence where variable DB stars (He-rich atmospheres) are found, and is bright enough ($V = 22.08$) that it could be monitored from the ground with a 4-m class telescope. In Figure 7 we present a finding chart for this object. The ZZ Ceti instability strip occurs near $(U - I) = -0.4$ or $(V - I) = 0.0$ which corresponds to $V \sim 24.1$ in M4. This is probably too faint for a ground-based search for variability with existing telescopes.

As a further test of the location of WDs in the cluster CMD, and inferentially as a check on the value of R_V and the reddening used, we display in Figure 8 the M_V , $(V - I)$ CMD for data from the four annuli. Again, these data have been selected to have errors in their magnitudes of ≤ 0.25 and χ values ≤ 1.5 . In the companion diagram we overlay the theoretical cooling curve for $0.5M_\odot$ DA WDs in these colors. As was evident in Figure 6, the agreement between the observed and theoretical loci is superb.

5. The White Dwarf Masses

The mean mass and the distribution of masses among the cluster WDs provide sensitive tests of stellar evolution theory and constrain the fraction of binaries in the cluster. For example, the mean WD mass is set by the electron degenerate core mass at the termination

of the asymptotic giant-branch phase of evolution and is predicted to be $0.53(\pm 0.02)M_{\odot}$ for stars currently terminating their evolution in a typical globular cluster (Renzini *et al.* 1996, Renzini & Fusi Pecci 1988). WDs with masses higher than this mean value could have originated from more massive progenitors, such as blue stragglers (BS), some of which are thought to be merged binaries. Good reading on BSs in globular clusters can be found in articles by Sarajedini, and Baily and Mader in "Blue Stragglers" (1993). Lower mass WDs could have evolved from mass transfer binaries in which a red giant with a predominantly helium core transfers mass to a companion and interrupts its evolution prematurely. Thus, the relative number of WDs with masses significantly different than the predicted mean of $0.53M_{\odot}$ may be related to the fraction of binaries present in the cluster. An estimate of the binary fraction involving non-interacting WD/red dwarf pairs will be given in §7.

We illustrate how low- or high-mass WDs could be recognized in the CMD by replotting in the left-hand panel of Figure 9 the M_U , $(U - I)$ CMD of Figure 6 together with cooling curves for 0.4 to $1.0M_{\odot}$ WDs in increments of $0.2M_{\odot}$. These cooling sequences are for pure carbon core evolutionary models with helium layers having 1% of the mass of the star and thick hydrogen layers of 0.01% by mass. The atmospheric colors are for DA models and are taken from Bergeron *et al.* (1995). The core composition makes little difference to the location of these cooling curves in the CMD (only the cooling times to a given luminosity are affected), but the composition of the atmosphere does. We discuss this latter point below.

To estimate the mean of the WD mass distribution, we fit a grid of cooling curves for DA WDs to our sample of WDs using the standard χ^2 goodness-of-fit statistic. The observed white dwarf colors were weighted by $w = 1/\sigma_s^2$, where σ_s is the frame-to-frame scatter in the magnitudes determined by ALLFRAME. The $M_{WD} = 0.51M_{\odot}$ DA cooling curve gave the best fit to the data in both the $U, U - I$ and $V, V - I$ planes. In order

to determine the uncertainty in this mass estimate we did a series of 1001 bootstrap resamplings of the data using standard bootstrapping techniques (*e.g.* Hill 1986, Feigelson & Babu 1992). That is, we took the original sample of 109 WDs (the number of WDs in Figure 6) and randomly picked, with replacement, 109 WDs from this sample. We then did a χ^2 fit to the resampled data in exactly the same manner as we did to the original data to determine the best-fitting DA cooling curve mass. The distribution of masses obtained from 1001 bootstrap resamplings was consistent with a Gaussian distribution. The mass distributions obtained from bootstrapping the data in each plane had means of $\overline{M}_{WD} = 0.507 \pm 0.025 M_{\odot}$ in the $U, U - I$ plane and $\overline{M}_{WD} = 0.510 \pm 0.023 M_{\odot}$ in the $V, V - I$ plane. The quoted uncertainties are one standard deviation. These results suggest that our data are consistent with the WDs in M4 having a single mass of $0.51 M_{\odot}$ with a $1\text{-}\sigma$ uncertainty of $0.03 M_{\odot}$. In contrast to the mean mass for these globular cluster WDs, the mean of the field WD mass distribution is somewhat higher at $0.59 M_{\odot}$ (Bragaglia, Renzini & Bergeron 1995). This is not surprising considering that the field sample is likely to have evolved, on average, from more massive progenitors which leave higher mass remnants.

The above result for the mean WD mass is likely to be somewhat in error as the possible presence of non-DA stars was not considered. The effect of a component of DB WDs, for example, on the observed WD sequence can be judged from the right-hand panel of Figure 9, where we illustrate the locus of $0.5 M_{\odot}$ cooling DA and DB stars. If DB WDs are in fact present in the cluster with the same ratio as in the field where about 20% of field WDs are of type DB (Sion 1986), then they will produce an apparent tail to higher masses if the WDs are analysed under the assumption of a pure DA sample. This is not a problem above a temperature of about 30,000 K where there are no DB WDs but, unfortunately, no WDs at this high a temperature are present in our sample. However, as can be seen in Figure 9, DAs and DBs are almost indistinguishable in the $V, V - I$ plane. The separation between the DA and DB curves is of order or smaller than the dispersion along the cooling

sequence. The good agreement between the mean masses derived in the two CMDs then argues that DBs have not been a serious problem.

Had we chosen, instead, to adopt a mass for the M4 WDs based on some initial-final mass relation, we could then have used the WDs to derive an accurate cluster distance as Renzini *et al.* (1996) have done for NGC 6752. If we choose, as they did, a mass of $0.53M_{\odot}$ for the WDs in a globular cluster and fit a cooling sequence of this mass to the observed WDs, we would then derive a distance modulus to M4 of $(m - M)_V = 12.60$. This is to be compared with 12.51 derived from the main sequence.

6. The White Dwarf Luminosity Function

Thus far we have shown that WDs are present in globular clusters and that they are located in the expected regions of the CMD, or, alternatively, that they possess about the masses predicted by theory. With the luminosity function we explore whether WDs are present in the expected numbers. This provides a sensitive test of the rate at which the WDs are cooling and thus of the evolutionary models, which, except for the preliminary analysis in Richer *et al.* (1995), have never actually been tested against a sequence of cooling WDs. The only other possible, similarly sensitive, test is to look for photometric period changes in variable WDs caused by cooling (Kepler *et al.* 1991, Bradley *et al.* 1992). These effects are sufficiently small that definitive results are still lacking for variable DA ($T_{\text{eff}} \sim 12,000$ K) and DB ($T_{\text{eff}} \sim 27,000$ K) WDs. To test the theory in detail, we develop the WD cumulative luminosity function (CLF) and compare it with theoretical luminosity functions constructed with three different core compositions, namely, pure carbon, mixed carbon and oxygen, and pure oxygen.

The CLF is built by counting WDs as a function of magnitude in each annulus and

correcting the star counts for incompleteness. It is clear from Figures 6 and 8 that no large corrections are required for either foreground or background objects. There may be a few contaminating WDs contributed from the Galactic bulge region. These objects should begin at an absolute V -magnitude near +13 in Figure 8 (the Galactic bulge possesses a distance modulus that is about 3 magnitudes larger than that of M4), but we estimate that their numbers must be no more than a few percent of the cluster WDs given the paucity of red giants contributed by the bulge in this diagram. Background blue galaxies that appear stellar are also rare. In the Hubble Deep Field, Reid *et al.* (1997) find two blue ($V - I \leq 1.0$) stellar objects, both with $V < 21$ and thus brighter than the WD sequence seen in Figure 8. In addition they find three $V \sim 27$ blue objects which *might* be stellar. Because of the apparently small contribution to the cluster WD sequence, we have chosen to ignore corrections for either foreground or background objects.

The incompleteness corrections to the counts were obtained by adding stars with WD colors and magnitudes along a dispersionless distribution (see Figure 10) to random locations in the frames and rereducing the frames to obtain the recovery statistics for the WDs. The WDs were added in small numbers (typically 150 per frame) so that they would not seriously affect the crowding statistics in the images. In total, almost 16,000 artificial WDs were added in more than 100 trials and the input and output sequences are illustrated in Figure 10. The CLF was constructed without binning as each counted star was simply replaced by n stars, where n is the inverse of the completeness fraction at that magnitude along the WD cooling sequence. Figures 11a (for the WF chips) and 11b (PC chips) display these incompleteness corrections in the four annuli in V for stars that are found on *both* V and I . In the analysis which follows, we set the magnitude limit for inclusion in the WD luminosity function to the point where the completeness in the sample drops to 30%. Determining these corrections in ALLFRAME is extremely cpu intensive as the WDs are added to *all* of the individual frames which are then processed exactly as the original

frames. About 6 cpu months on a SPARC 20 were required to complete this analysis. Further details relating to the procedures employed can be found in Ibata *et al.* (1997).

Figure 12 illustrates the observed CLF for the WDs in the four annuli. The data extend to deeper magnitudes in more distant annuli mainly due to the decreasing effects of crowding and scattered light from saturated stars. Interpreting Figure 12 requires comparing the observed CLFs with those derived by combining theory and the number of WD progenitors in each field. However, what can be said is that the slope of the differential luminosity function ($d[\log N_{WD}]/d[M_V]$) for the cluster WDs in annulus 4, 0.29 ± 0.07 , is consistent with the 0.39 ± 0.05 found for a field sample (Winget *et al.* 1987, Liebert *et al.* 1988, Oswalt *et al.* 1996).

We compare the observed CLF with theory by initially making the assumption that the number of stars in any region of the cluster is conserved as they move through their post-main-sequence phases of evolution. This does not allow for evaporation or tidal stripping of stars from the cluster or, more importantly, does not take into consideration mass segregation (see Fahlman *et al.* 1996b for a detailed discussion of this point). The hypothesis of a conserved number of stars will become increasingly less valid the older the WD because the cooling time of the star can become longer than the relaxation time of the cluster. However, the obvious first step is to compare the number of WDs present in some field with that of a tracer population with a well-defined mass and lifetime. With such an approach, the effect of mass segregation becomes strictly a differential one due to the difference in mass between the WDs and the tracer. A convenient stellar population for this purpose is the cluster horizontal branch (HB) stars.

If there is no differential mass segregation between HB and WD stars, then the numbers of stars in the two post-main sequence phases will be strictly determined by the ratio of the

lifetimes of those phases. This yields

$$N_{WD}(< M_V) = N_{HB} \cdot \left(\frac{t_{cool}(M_V)}{t_{HB}} \right) \quad (1)$$

where $N_{WD}(< M_V)$ is the number of WDs brighter than absolute magnitude M_V , N_{HB} is the number of HB stars, t_{cool} the WD cooling time to reach M_V , and t_{HB} the lifetime of a star on the HB, which we take as 10^8 yr (Renzini 1977, Dorman 1992). All of the WD theory is in t_{cool} , which comes from Wood’s (1995) models together with the model atmospheres of Bergeron *et al.* (1995). N_{HB} , the number of HB stars within the area of each annulus covered by our *HST* observations, was estimated from ALLFRAME photometry of a 300 sec *B*-image and a 160 sec *V*-image taken (by C. Pryor) with a Tektronix 2k CCD on the 0.9-m telescope at Kitt Peak. Since the number of HB stars is always small (there are only ~ 150 HB stars in the entire cluster), we measured the numbers of giant branch (GB) stars with $V < 16$ and HB stars at different distances from the cluster center and used the average ratio of 0.135 HB stars/GB star to set the HB number. Both the GB and HB stars are bright enough that their numbers have not been affected significantly by incompleteness. This ratio represents a global average as it is determined from data extending well past the half mass radius for which the relaxation time is too long to have caused significant mass segregation between the HB and GB stars. Because we do use the number of GB stars to represent the HB population, our HB stars are distributed in the cluster *as if* they possessed masses of $0.81M_\odot$, which is the mass of turnoff stars in M4 (Bergbusch & Vandenberg 1992).

We cannot expect that agreement between the theory and observations will be perfect as the sample of normalizing stars is not large, differential mass segregation between the ($0.81M_\odot$) HB stars and the ($0.51M_\odot$) WDs has not been accounted for, and the WD population is modest. In the first comparison between theory and observations (Figure 13) we use models with pure carbon core interiors together with DA atmospheres. While the

similarity between theory and observation is apparent, there remains a systematic trend in the normalization between the two. The normalization is too high in the inner annuli (too many HB stars compared to the number of WDs) and too low in the outer ones (too few HB stars). This is exactly what is expected from mass segregation (see Fahlman *et al.* 1996b).

We correct for differential mass segregation by fitting the projected density profile of the GB stars to the GB mass profile in a multi-mass Michie-King (1966) model. This fit yields the total number of such stars in the cluster. We then calculate the projected density profile of a $0.51M_{\odot}$ component in the model that has the same total number of stars. This assumes that the WDs have come into equilibrium (as defined by a King model) at all radii. This is somewhat extreme, as the relaxation time at large radii can exceed the age of the cluster. Using the surface density of the $0.51M_{\odot}$ component, we then predict the number of HB star progenitors for each of the four WD CLFs by multiplying by the area observed and $\text{HB/GB} = 0.135$. This result is only very weakly dependent on the slope of the cluster mass function used in the model; for a slope of $x = 0.7$ (Salpeter = 1.35), we obtain the normalizations indicated in Figure 14. Here the agreement is much improved over Figure 13: only the CLF in annulus 2 appears to deviate significantly from the theoretical expectations. However, even in this case the difference at the faint end only amounts to about a $1.5\text{-}\sigma$ deviation, a difference that is seen by chance 13% of the time. Figure 14 illustrates that when the observed amount of mass segregation between the WDs and the GB stars is taken into account, the CLFs provide a strong positive test of the correctness of current WD cooling theory.

The analysis presented above is strictly correct only for WD cooling times, t_{cool} , that are small compared to the age of the cluster. If the cooling times are an appreciable fraction of the cluster age, the slope of the cluster initial mass function (IMF) and the relation between stellar mass and main sequence lifetime become important. For example, if we

choose a global power-law IMF of the form $n(m) \propto m^{-(1+x)}$ and the current main-sequence lifetime as a function of mass is represented by $t_{MS} \propto m^{-\gamma}$, then in the limit of long cooling times ($t_{cool} \rightarrow \text{age of the cluster}$),

$$N_{WD}(< M_V) = N_{HB} \cdot \left(\frac{t_{cool}(M_V)}{t_{HB}} \right) \cdot \left(\frac{\gamma}{x} \right). \quad (2)$$

Reasonable values for x and γ are 0.7 and 2.5, respectively, so that the expected number of faint WDs can potentially be increased by up to a factor of three.

We can repeat the analysis displayed in Figure 14 with core compositions consisting of a mixture of carbon and oxygen or of pure oxygen (evolutionary models with these compositions have been calculated by Wood 1995), and ask if the comparison with theory is similarly satisfactory. WDs will cool at different rates with these varying core compositions since the heat capacity of the atoms differ. For example, a WD with a pure oxygen core is younger than one with a pure carbon core at a given luminosity. This, then, is potentially an effective way of determining the core composition of these objects. In Figure 15 we compare our observations to Wood’s models for pure oxygen, which is a more dramatically different core composition than the carbon-oxygen mixture. As can be seen, the results differ little from those in Figure 14: small changes in the normalization will make the observations fit the data as well as they do for the carbon-core models. The conclusion must then be that the sample of WDs itself and/or the number of normalizing HB stars remain too small, or that the test is simply too insensitive to use as a core chemical composition indicator.

Theory could potentially shed some light on the WD composition. Post-asymptotic giant branch evolutionary models of several researchers have agreed for a number of years on the general trends in the C/O profiles of the WD remnants they produce: an O-rich inner core, surrounded by a large transition zone which tails off to a nearly-pure C outer core (*cf.* Mazzitelli & D’Antona 1986 with Blöcker 1995). However, these profiles are sensitive to the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate, which has proved difficult both to calculate

and to measure (Azuma *et al.* 1994). This inability to distinguish cleanly the chemical make-up of the core has a modest effect on using the WD CLF as an astronomical clock. The resulting uncertainties in composition translate to $\sim 5\%$ uncertainty in the ages.

7. Binaries Among the White Dwarfs

Are a significant fraction of WDs tied up in binary systems with main sequence stars? The binary fraction is of general interest in dynamical studies of globular clusters since heating from close binaries is expected to inhibit, delay, or even reverse core collapse in these systems (Hut *et al.* 1992). Detailed discussions of the fraction of binaries among M4 main-sequence stars can be found in Fahlman *et al.* (1997), Pryor *et al.* (1996) and Côté & Fischer (1996).

We can use the color-color diagram of the cluster to investigate the frequency of binaries involving stars of very different colors (see Richer *et al.* 1996b). The system of a red dwarf star in a binary with a WD will have a rather peculiar color. In Figure 16 we plot the color-color diagram for all of the stars measured in three colors (annuli 1-3), and we include the results from a simulation that draws 100 stars at random from the WD sequence and the main sequence and then determines the color of the resulting object. Main sequence stars were chosen only over the same magnitude range as the WDs so no very bright red dwarfs went into the simulation. The one realization of this shown in Figure 16 (plotted as triangles) illustrates the location of WD/red dwarf binaries in the color-color plane. About 50% of the simulated binaries lie very close to the main sequence as in these cases the color is dominated by that of the red dwarf. In the regime where the simulated binaries are well removed from the main sequence, examination of Figure 16 indicates that ≤ 15 of the M4 stars are good candidates for such binaries. Given the crowding statistics on the frames, as judged by the artificial star tests, very few if any of these should be optical

(i.e., non-physical) systems. Thus, with more than 1000 main sequence stars in Figure 16 covering the same magnitude range as the WDs, less than 2% of all such objects appear to be involved in binary systems with a WD. This number could be higher by about a factor of two or so in order to account for systems lying too close to the main sequence to be distinguished from single stars.

8. Extending the Cluster Main Sequence Mass Function

Up to what mass does the IMF of a globular cluster extend? The answer is currently unknown, but it is certain to extend at least up to main sequence masses required to produce neutron stars as, at last count, 34 pulsars had been found in globular clusters (see e.g. Lyne *et al.* 1996). It is important to establish this upper limit as the dynamical history of the cluster is to a great extent controlled by its IMF. Possible self-contamination of the proto-cluster cloud with metals will also depend on the number of massive stars originally formed, with differing star formation scenarios in the early Universe predicting diverse values for the massive star fraction.

According to the M4 photometry of Richer & Fahlman (1984) and the models of Bergbusch & Vandenberg (1992), turnoff stars in M4 possess masses of $\sim 0.81M_{\odot}$. Thus from an examination of the mass function of present-day hydrogen-burning stars in this cluster, we learn little about the massive stars initially present. The cluster WDs, having evolved from stars more massive than the current cluster turnoff, provide the potential of extending the mass function to heavier objects: the older the WD, the more massive its progenitor.

As we will see in §9, the oldest WDs currently observed in the cluster have ages ~ 9 Gyr. If the cluster is 15 Gyr old, then these WDs evolved from stars with main-sequence

lifetimes of ~ 6 Gyr. Such stars have initial masses of $\sim 1.2M_{\odot}$, so the current data potentially allow us to extend the cluster mass function by about $0.4M_{\odot}$. However, the reality is that we are not confident in the completeness corrections when they fall below 30%, which limits the oldest WDs for which we have convincing statistics to an age of only 3.7 Gyr. The progenitor masses in this case are $\sim 0.9M_{\odot}$. Hence we can confidently extend the M4 mass function by only $0.1M_{\odot}$. We carried out this exercise and found that the extension of the M4 mass function derived by Fahlman *et al.* (1997) up to $0.9M_{\odot}$ appears to be rather flat ($x = -1$). However, this likely does *not* represent the IMF, as neither the loss of stars from the cluster nor mass segregation have been considered here. These are likely to have an important influence on the cluster mass function as its small w velocity (Cudworth & Rees 1990b) suggests that M4 spends most of its orbit near the Galactic disk where shocking and tidal effects can efficiently remove low mass stars from it.

There is, however, the potential of using the WD CLF to explore the IMF of a globular cluster to much higher masses. The dependence of $N_{WD}(< M_V)$ on the IMF slope x is explicitly given in equation 3, which is correct to second order in the WD cooling times and for which equations 1 and 2 are limiting cases:

$$N_{WD}(< M_V) = N_{HB} \cdot \left(\frac{t_{cool}(M_V)}{t_{HB}} \right) \cdot \left(1 + \frac{t_{cool}(\gamma - x)}{2\gamma t_O} \right). \quad (3)$$

Here t_O is the current age of the cluster. While this expression is dependent on the assumption of star conservation, stellar losses and mass segregation could, in principle, be accounted for. Deviations from a power-law slope in the CLF would then contain information on the cluster IMF. We explicitly demonstrate the effect on the WD CLF of including the mass spectrum and variation in main sequence lifetime with mass by replotting in Figure 17 the data from annulus 4 and using equation 3 to represent the theory. The effect of including the cluster mass function slope is to decrease the number of WDs (as x is generally positive this produces fewer high mass stars that eventually become

WDs). The inclusion of the main sequence lifetimes acts in the opposite sense, as the more massive stars spend a shorter time on the main sequence and hence produce WDs more quickly. For $x = +2.5$ and $\gamma = 2.5$ the two contributions cancel each other. Figure 17 illustrates that over the extreme range of x for fixed γ , the expected number of WDs to an $M_V = +16$ will vary by about 40%. Taking Figure 17 at face value, it appears that under the unlikely assumption of no loss of stars from the cluster, the IMF of M4 must be rather steep if γ is in the reasonable range of 2.5. Allowing for the depletion of old WDs through some star-loss mechanisms suggests that a flatter IMF is allowed.

9. White Dwarfs and the Age of the Universe

There is currently an unexplained difference in the ages of the Universe derived from the expansion and from its oldest datable component, the globular clusters. With $H_O = 70$ km/sec/Mpc (a median value from the recent Space Telescope Science Institute workshop on the distance scale), standard inflationary cosmology ($\Omega = 1, \Lambda = 0$) yields 9.5 Gyr for the expansion age, and hence $\lesssim 9$ Gyr for the age of the oldest globular clusters. By contrast, globular cluster ages, derived via the luminosity of the main-sequence turnoff, are in the range of 14-16 Gyr (Richer *et al.* 1996a, Chaboyer *et al.* 1996, Bolte & Hogan 1995). While it is possible to reconcile these two ages with non-standard cosmologies (e.g., $\Lambda \neq 0$), such a solution will not be compelling until all reasonable avenues to check these ages are explored. A new approach to the globular cluster ages can come from the ages of their coolest WDs. The age of a WD is directly related to its luminosity, and the time for a typical WD to cool to invisibility exceeds any reasonable estimate for the age of the Universe. Thus, a cutoff in the WD luminosity function at low luminosity will provide the time since the formation of the first cluster WDs, and hence a lower limit to the age of the Universe.

The accuracy of WD cooling theory will be an important issue in this discussion. To

the confidence expressed in §7 in the current models, we can add the following points. The most ancient WDs included in annulus 4 of Figure 14 are almost 4 Gyr old. Older WDs are measured but are not included as the incompleteness corrections for such faint objects are too insecure. Crystallization of the core of a $0.5M_{\odot}$ WD begins when it is only 3 Gyr old, so the data demonstrate that the models seem to be reliable even into this regime where the physics of the interior is more complex. Further, in the derivation of the age of the disk of the Galaxy using WDs (Winget *et al.* 1987), much cooler WDs were used. In these older stars almost the entire core of the WD has crystallized, yet the slope of the WD luminosity function down to at least 9 Gyr agreed well with the theory.

To explore the ages of the WDs currently seen in M4, we plot in Figure 18 the V and I data selected to have errors in their magnitudes of ≤ 0.5 and χ values ≤ 2.0 . These are more liberal criteria so this CMD will contain fainter and more poorly measured stars than the one in Figure 8. The right-hand panel of the Figure overlays a theoretical cooling curve for $0.5M_{\odot}$ carbon core DA WDs that extends to an age of 13 Gyr. The most ancient WDs that are currently seen in M4 are ~ 9 Gyr old. If we take this as a strict lower limit to the age of the Universe then *this result, by itself, requires $H_0 < 73$ in standard cosmological models with $\Omega = 1$* . These oldest WDs are within about 1 Gyr of the most ancient ones currently known in the disk of the Galaxy (Oswalt *et al.* 1996, Ruiz *et al.* 1995, Winget *et al.* 1987).

The future use of WDs in probing cosmological models will come from data sets which penetrate to equivalent V magnitudes in M4 of 30 or more ($M_V \geq 17.5$). At this level, the WDs are 12 Gyr or older and begin to provide interesting constraints. Photometry at such faint magnitudes can be obtained in about 100 orbits with the current instrumentation on *HST*, and in significantly less time when the Advanced Camera is available after 1999. It may also possibly be obtained in the infrared with ground-based 8-10 meter class telescopes

equipped with adaptive optics.

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FIGURE CAPTIONS

FIG. 1.— The location of all the stars measured in M4 with respect to the cluster center. The 5 radial distances indicated are at $0.5, 1.5, 2.5, 4.0$ and $8r_c$ where $1r_c$ is taken to be $50''$. The decrease in the surface density of stars with increasing distance from the cluster center can be clearly seen in this diagram.

FIG. 2.— A comparison between ground-based V and I photometry from the Las Campanas 2.5 meter telescope with that from *HST*. All the stars shown are in annulus 4.

FIG. 3.— The U , $(U - V)$ M4 fiducial main sequence based on Las Campanas and CTIO photometry (dashed line) compared with that from *HST* (solid line).

FIG. 4.— Best fit of a sample of subdwarfs (Mandushev *et al.* 1996) to the ground-based fiducial main-sequence of M4 (solid line). The subdwarf colors were adjusted to simulate stars with the same metal abundance as that of the cluster, $[\text{Fe}/\text{H}] = -1.33$.

FIG. 5.— Contours of equal χ^2 representing the goodness-of-fit between the subdwarfs and the M4 main-sequence fiducial. The contour interval is 1.1 in $\log(\chi^2/\eta)$, where η is the number of degrees of freedom (8 in our case). Fitting the subdwarfs in this manner clearly constrains the cluster reddening rather well, but does a less good job of constraining the cluster distance modulus. The minimum χ^2 , indicated by the plus sign, is obtained for $E(V - I) = 0.51$ and $(m - M)_V = 12.51$, which are the values we adopt for M4.

FIG. 6.— The M_U , $(U - I)$ CMD for stars in the inner 3 annuli. All stars present in this diagram have $\chi < 1.5$ (a measure of the goodness-of-fit between the star and the point spread function [Stetson 1987]) and errors in photometry (judged from measurement of the same star on numerous individual frames) less than 0.25 mags. The right hand panel plots the same data but includes a $0.5M_\odot$ hydrogen-rich WD cooling sequence derived as

described in the text.

FIG. 7.— Finding chart for the hot WD found in M4. The axes in the plot are in arc seconds and north and east are indicated. Coordinates (2000.0) for the WD are $RA = 16^h23^m38^s.66$, $DEC = -26^\circ32'10''.90$. Pixel coordinates on our frame are $(x,y) = (310.042, 375.391)$.

FIG. 8.— The M_V , $(V - I)$ CMD for stars in all four annuli. All stars present in this diagram were selected as in Fig. 6. The right-hand panel plots the same data but includes a $0.5M_\odot$ hydrogen-rich WD cooling sequence derived as described in the text.

FIG. 9.— As in Figure 6, but in the left hand panel a sequence of DA WD cooling curves have been overlaid. This sequence runs from $0.4M_\odot$ to $1.0M_\odot$ in increments of $0.2M_\odot$ and can be used to determine the mean mass of the cluster WDs. In the companion panel, $0.5M_\odot$ DA and DB cooling curves illustrate how an admixture of WD types can potentially affect the mass estimates.

FIG. 10.— The left panel is the dispersionless sequence of stars that was input into the M4 data frames in groups of 150. These stars were then photometered exactly as the real stars. Note that both WDs and main sequence stars were added; however, here we are concerned only with the WDs. The right panel shows the output of this exercise. The recovery statistics of these added stars are then used to estimate both the completeness of the counted stars as well as their uncertainties.

FIG. 11(a).— The completeness percentages in V of added WDs on the WF chips in the four annuli. These numbers are for stars with WD colors, thus the percent complete represents the probability of recovering an input star at the indicated V magnitude with WD colors on *both* the V and I frames.

FIG. 11(b).— Same as (a), except for the PC chips.

FIG. 12.— The cumulative WD luminosity function (CLF) in the four annuli. The CLF in the annuli more distant from the cluster center penetrate to fainter magnitudes mainly due to the effects of their being fewer saturated stars present in the frames.

FIG. 13.— As in Figure 12, except that a theoretical CLF (dotted line) is overlaid on the data. In this case, the theoretical sequence is for DA WDs with pure carbon cores. The normalization of the theoretical CLF is expressed in terms of the number of HB stars, N_{HB} , expected in the area of the annulus covered by the *HST* observations. Theoretical CLFs derived using the $\pm 2\sigma$ errors on N_{HB} are plotted as dashed lines, clearly illustrating that the differences seen between the observations and the theory are highly significant. Mass segregation *has not* been accounted for in this diagram.

FIG. 14.—As in Figure 13, except that mass segregation *has* been accounted for in the normalization of the theoretical CLF with the HB stars. This procedure is described in detail in the text.

FIG. 15.— As in Figure 14 except that the CLF is for DA WDs with pure oxygen cores. Note that the fit between the observed and theoretical CLFs differ little from those in Figure 14 and that a small adjustment in the number of HB stars will make the comparison as good as that for the pure carbon core models. This demonstrates that the core composition of globular cluster WDs is unlikely to be determined with this technique.

FIG. 16.— The $(U - V)$, $(V - I)$ color-color diagram for stars in the inner three annuli wherein F336W exposures were obtained. No reddening corrections were applied. The cluster WDs, plotted as plus signs, are the objects with $(U - V) < 0.7$. Main-sequence stars are plotted with small dots and the location of 100 artificial WD/red dwarf binaries are shown as filled triangles. A few M4 objects are possible candidates for such binary systems, but when the probability of optical binaries is included, the number is small.

FIG. 17.— The observed WD CLF in annulus 4 compared with theoretical luminosity functions wherein the effect of the IMF and the dependence on mass of the main sequence lifetime of a star are included.

FIG. 18.— M_V versus $(V - I)$ for stars in all of the annuli that have photometric errors < 0.5 mags. in each color and $\chi < 2$. An extended DA WD cooling sequence for C-core composition is included in the right panel to indicate the ages of the cooling WDs. WDs as old as 9 Gyr appear to have been detected in M4.

Table 1. M4 White Dwarfs

F	C	X	Y	RA	DEC	U	$\sigma(U)$	V	$\sigma(V)$	I	$\sigma(I)$
0	1	496.143	344.045	245.90711	−26.52547	25.807	1.344	24.084	0.053	23.576	0.057
0	1	362.712	469.314	245.90763	−26.52320	26.342	1.049	22.112	0.035	21.873	0.037
0	1	603.582	677.165	245.90318	−26.52282	26.322	0.767	24.255	0.047	23.930	0.076
0	1	380.025	67.895	245.91069	−26.52746	25.294	0.391	24.457	0.043	24.028	0.140
0	1	737.100	133.400	245.90606	−26.52937	26.277	0.059	26.086	0.173	24.993	0.177
0	1	351.673	188.520	245.91004	−26.52602	26.155	0.381	25.256	0.093	24.559	0.172
0	1	494.198	227.122	245.90809	−26.52666	24.676	9.999	24.907	0.052	24.167	0.032
0	1	531.399	230.733	245.90762	−26.52689	25.774	0.941	24.449	0.064	23.688	0.201
0	1	368.113	292.171	245.90901	−26.52507	26.385	9.999	25.496	0.100	24.532	0.081
0	1	49.368	304.191	245.91256	−26.52263	25.821	0.779	25.395	0.116	24.532	0.102
0	1	192.136	436.617	245.90986	−26.52230	25.666	0.596	24.130	0.054	23.691	0.062
0	1	778.726	623.622	245.90161	−26.52465	26.945	0.374	26.553	0.291	25.326	0.396
0	1	280.881	643.384	245.90716	−26.52082	25.765	1.268	25.189	0.094	24.351	0.109
0	1	578.181	520.223	245.90473	−26.52425	25.725	2.473	24.689	0.080	24.071	0.143
0	2	475.710	108.076	245.90911	−26.51432	23.550	0.072	23.734	0.107	23.427	0.099
0	2	322.573	187.614	245.91384	−26.51647	24.117	0.235	23.827	0.090	23.442	0.100
0	2	670.463	265.761	245.90956	−26.50742	23.004	0.152	22.899	0.074	22.401	0.060
0	2	764.570	388.489	245.91093	−26.50335	24.304	0.201	23.919	0.107	23.399	0.077
0	2	184.601	33.322	245.91249	−26.52201	24.253	0.232	24.052	0.131	23.531	0.174
0	2	430.132	173.199	245.91156	−26.51429	26.418	1.594	26.124	0.230	24.906	0.270
0	2	548.990	306.412	245.91275	−26.50947	26.155	1.868	26.261	0.414	24.820	0.258
0	2	767.149	326.185	245.90934	−26.50429	26.497	0.580	26.107	0.263	25.015	0.410
0	2	621.544	336.663	245.91220	−26.50736	26.898	0.394	26.792	0.442	25.878	0.329
0	2	616.555	341.072	245.91239	−26.50740	25.385	0.426	24.956	0.127	24.150	0.149
0	2	655.539	379.733	245.91267	−26.50591	25.706	0.479	25.442	0.106	24.701	0.088

Table 1—Continued

F	C	X	Y	RA	DEC	U	$\sigma(U)$	V	$\sigma(V)$	I	$\sigma(I)$
0	2	537.197	381.248	245.91484	−26.50853	26.597	0.853	26.681	0.294	25.253	0.202
0	2	685.570	456.725	245.91406	−26.50400	25.769	0.613	25.675	0.187	24.799	0.186
0	2	744.612	524.205	245.91468	−26.50161	25.867	0.519	26.090	0.315	25.074	0.210
0	2	679.611	525.742	245.91589	−26.50302	25.670	0.540	25.233	0.138	24.361	0.122
0	2	784.167	542.302	245.91442	−26.50045	24.809	0.249	24.665	0.135	24.160	0.156
0	3	412.694	713.550	245.93618	−26.53406	23.112	0.117	23.166	0.065	23.007	0.067
0	3	596.394	779.435	245.94194	−26.53258	21.885	0.066	22.415	0.040	22.161	0.023
0	3	506.072	717.885	245.93859	−26.53266	25.028	0.280	24.490	0.080	23.703	0.112
0	3	479.387	782.073	245.93907	−26.53451	25.904	0.762	26.154	0.188	25.156	0.190
0	4	131.966	322.229	245.90930	−26.53149	24.240	0.183	23.926	0.092	23.281	0.038
0	4	550.987	339.503	245.91625	−26.54125	23.776	0.208	23.735	0.046	23.392	0.048
0	4	109.239	359.538	245.90795	−26.53157	23.758	0.105	23.711	0.094	23.417	0.138
0	4	310.042	375.391	245.91108	−26.53636	21.630	0.056	22.075	0.063	21.938	0.043
0	4	258.894	473.253	245.90771	−26.53675	23.045	0.148	23.340	0.101	23.108	0.084
0	4	431.768	475.534	245.91070	−26.54071	23.356	0.163	23.411	0.073	23.154	0.093
0	4	410.933	740.087	245.90365	−26.54440	24.268	0.125	24.102	0.068	23.653	0.135
0	4	712.836	235.416	245.92172	−26.54326	26.817	9.999	27.181	0.442	25.796	0.226
0	4	771.054	315.314	245.92073	−26.54583	25.963	0.346	26.547	0.305	25.309	0.467
0	4	721.709	349.379	245.91900	−26.54526	26.397	0.752	26.578	0.239	25.091	0.322
0	4	647.282	421.195	245.91587	−26.54472	27.272	0.903	26.618	0.498	25.064	0.400
0	4	576.599	471.804	245.91336	−26.54393	24.942	0.413	24.267	0.107	23.535	0.094
0	4	635.669	472.746	245.91438	−26.54527	25.967	0.547	25.619	0.221	24.793	0.243
0	4	589.519	496.091	245.91297	−26.54460	26.534	0.323	26.373	0.226	25.132	0.250
0	4	784.239	498.760	245.91632	−26.54901	26.384	0.771	26.065	0.240	25.034	0.364
0	4	635.871	499.790	245.91369	−26.54570	25.769	0.494	25.495	0.117	24.597	0.112

Table 1—Continued

F	C	X	Y	RA	DEC	U	$\sigma(U)$	V	$\sigma(V)$	I	$\sigma(I)$
0	4	632.871	593.176	245.91128	−26.54710	27.376	4.721	26.917	0.452	24.983	0.220
0	4	643.786	600.402	245.91128	−26.54746	26.015	1.169	25.629	0.218	24.792	0.128
0	4	686.579	782.843	245.90745	−26.55125	24.276	0.225	24.029	0.062	23.618	0.093
0	4	289.603	784.965	245.90040	−26.54236	22.583	0.076	22.935	0.060	22.687	0.062
0	4	432.300	516.846	245.90966	−26.54137	25.587	0.502	25.027	0.120	24.149	0.316
0	4	312.866	587.359	245.90577	−26.53978	25.580	0.737	25.250	0.158	24.528	0.262
1	1	349.394	277.235	245.92100	−26.51353	25.723	0.475	23.235	0.044	22.866	0.049
1	1	430.304	291.978	245.91996	−26.51397	25.613	2.206	24.177	0.070	23.654	0.060
1	1	586.114	453.700	245.91687	−26.51344	26.501	0.476	23.824	0.059	23.414	0.069
1	1	522.663	486.096	245.91732	−26.51265	25.290	0.273	23.783	0.051	23.316	0.040
1	1	135.349	324.071	245.92308	−26.51150	26.413	0.124	24.031	0.051	23.611	0.070
1	1	475.811	78.225	245.92117	−26.51648	25.709	1.143	26.467	0.164	25.382	0.301
1	1	602.277	114.687	245.91943	−26.51702	25.362	1.257	26.027	0.134	24.800	0.134
1	1	673.158	205.174	245.91789	−26.51661	26.552	0.453	25.263	0.059	24.439	0.076
1	1	292.554	214.222	245.92216	−26.51377	26.225	0.697	26.081	0.146	25.045	0.187
1	1	615.985	328.506	245.91754	−26.51494	26.762	0.049	25.998	0.104	25.084	0.149
1	1	137.536	355.195	245.92280	−26.51120	25.660	1.500	24.766	0.060	24.111	0.041
1	1	593.278	364.668	245.91751	−26.51441	26.982	0.533	27.741	0.494	26.018	0.452
1	1	360.138	375.973	245.92008	−26.51260	25.629	0.849	25.194	0.082	24.466	0.106
1	1	564.521	389.121	245.91763	−26.51395	27.683	9.902	27.454	0.350	25.640	0.300
1	1	491.151	403.723	245.91835	−26.51327	26.275	1.458	23.930	0.053	23.410	0.044
1	1	126.262	604.305	245.92091	−26.50857	25.667	0.597	23.962	0.099	23.608	0.120
1	1	438.086	634.244	245.91709	−26.51052	26.880	0.931	27.631	0.341	25.989	0.414
1	1	695.747	750.784	245.91323	−26.51120	26.771	2.659	26.591	0.166	25.034	0.222
1	1	156.746	381.055	245.92237	−26.51107	26.489	0.732	27.275	0.244	25.944	0.394

Table 1—Continued

F	C	X	Y	RA	DEC	U	$\sigma(U)$	V	$\sigma(V)$	I	$\sigma(I)$
1	1	260.636	481.355	245.92036	−26.51079	25.197	0.551	24.711	0.080	24.017	0.085
1	1	748.344	639.979	245.91352	−26.51271	25.807	1.088	26.499	0.194	25.417	0.325
1	2	155.110	172.465	245.92811	−26.50892	23.150	0.079	23.342	0.056	23.010	0.055
1	2	594.307	231.544	245.92170	−26.49819	22.224	0.068	22.641	0.039	22.354	0.049
1	2	57.763	306.117	245.93318	−26.50894	24.109	0.190	24.034	0.126	23.462	0.063
1	2	373.536	347.314	245.92854	−26.50125	24.524	0.407	24.271	0.069	23.595	0.140
1	2	80.782	412.613	245.93542	−26.50672	24.278	0.117	24.324	0.072	23.710	0.095
1	2	111.330	598.374	245.93949	−26.50305	24.293	0.158	24.213	0.075	23.479	0.083
1	2	286.338	603.237	245.93649	−26.49908	22.315	0.096	22.625	0.045	22.267	0.029
1	2	324.737	629.303	245.93644	−26.49781	24.232	0.176	24.007	0.054	23.515	0.056
1	2	162.090	701.645	245.94113	−26.50027	22.840	0.145	23.111	0.110	22.695	0.119
1	2	788.443	794.648	245.93220	−26.48495	23.827	0.347	23.598	0.049	23.168	0.082
1	2	593.449	90.630	245.91822	−26.50047	23.683	0.155	23.490	0.081	22.930	0.065
1	2	361.919	78.734	245.92207	−26.50582	25.396	0.460	25.639	0.218	24.723	0.349
1	2	340.592	103.637	245.92308	−26.50590	25.498	0.521	25.494	0.143	24.548	0.115
1	2	714.394	137.017	245.91723	−26.49704	26.608	0.557	26.205	0.165	25.101	0.136
1	2	746.224	151.223	245.91702	−26.49611	23.867	0.222	23.957	0.080	23.466	0.117
1	2	216.170	172.611	245.92701	−26.50756	25.636	0.548	26.163	0.192	24.958	0.148
1	2	88.418	195.916	245.92990	−26.51002	25.039	0.322	24.451	0.114	23.968	0.139
1	2	115.700	226.671	245.93017	−26.50893	25.890	0.673	25.630	0.143	24.866	0.330
1	2	600.651	253.046	245.92211	−26.49770	25.658	0.453	25.415	0.108	24.705	0.177
1	2	97.713	283.199	245.93190	−26.50842	25.590	0.308	25.385	0.137	24.545	0.205
1	2	472.723	471.692	245.92987	−26.49704	25.390	0.289	24.992	0.114	24.202	0.199
1	2	156.768	483.856	245.93583	−26.50389	26.550	0.894	26.027	0.273	24.656	0.205
1	2	587.031	488.731	245.92823	−26.49422	26.150	0.456	26.136	0.322	24.889	0.200

Table 1—Continued

F	C	X	Y	RA	DEC	U	$\sigma(U)$	V	$\sigma(V)$	I	$\sigma(I)$
1	2	625.878	496.426	245.92773	−26.49323	26.184	0.911	26.446	0.304	25.476	0.327
1	2	349.245	511.384	245.93307	−26.49916	26.237	0.755	25.778	0.178	25.080	0.296
1	2	147.867	515.218	245.93678	−26.50358	24.803	0.125	24.669	0.102	24.312	0.109
1	2	756.205	631.259	245.92875	−26.48822	26.967	0.533	27.674	0.450	26.265	0.444
1	2	294.998	665.264	245.93786	−26.49789	27.245	2.730	28.364	0.413	26.042	0.495
1	2	237.385	725.273	245.94038	−26.49821	24.006	0.186	23.940	0.063	23.441	0.077
1	2	236.032	790.145	245.94200	−26.49720	26.302	1.116	26.099	0.231	24.887	0.233
1	2	334.299	108.831	245.92330	−26.50596	26.010	0.643	25.950	0.206	25.153	0.468
1	2	354.561	138.707	245.92369	−26.50503	25.350	0.362	24.970	0.147	24.212	0.137
1	3	439.010	92.945	245.93733	−26.50819	23.923	0.128	23.873	0.040	23.468	0.059
1	3	610.590	190.707	245.94335	−26.50764	23.541	0.083	23.557	0.038	23.175	0.067
1	3	594.596	224.457	245.94355	−26.50864	22.476	0.044	22.864	0.029	22.472	0.074
1	3	258.688	398.806	245.93826	−26.51789	23.164	0.104	23.356	0.056	23.055	0.071
1	3	745.151	490.305	245.95202	−26.51219	23.021	0.075	23.505	0.043	23.337	0.044
1	3	687.013	543.497	245.95153	−26.51430	22.828	0.033	23.089	0.035	22.819	0.049
1	3	69.732	550.776	245.93626	−26.52428	25.041	0.344	24.772	0.058	24.062	0.089
1	3	332.595	560.433	245.94299	−26.52033	24.600	0.276	24.163	0.054	23.671	0.097
1	3	192.072	567.610	245.93960	−26.52272	24.716	0.317	24.379	0.060	23.767	0.078
1	3	252.974	655.715	245.94270	−26.52372	23.281	0.116	23.435	0.035	23.091	0.078
1	3	732.149	718.742	245.95573	−26.51749	23.244	0.125	23.350	0.042	23.026	0.059
1	3	231.610	63.617	245.93164	−26.51084	23.217	0.152	23.297	0.053	22.859	0.042
1	3	382.830	108.107	245.93620	−26.50942	26.257	0.607	26.045	0.131	25.083	0.177
1	3	139.245	133.553	245.93058	−26.51387	26.527	0.687	25.929	0.165	24.946	0.264
1	3	503.679	134.166	245.93968	−26.50807	25.746	0.437	25.469	0.097	24.621	0.080
1	3	638.194	139.601	245.94313	−26.50607	24.949	0.138	24.762	0.071	24.208	0.160

Table 1—Continued

F	C	X	Y	RA	DEC	U	$\sigma(U)$	V	$\sigma(V)$	I	$\sigma(I)$
1	3	368.116	208.699	245.93760	−26.51189	26.795	0.196	26.326	0.269	25.493	0.365
1	3	365.608	256.783	245.93840	−26.51301	25.620	0.564	25.248	0.066	24.392	0.143
1	3	749.597	266.914	245.94818	−26.50714	25.522	0.528	25.218	0.084	24.510	0.162
1	3	660.734	332.531	245.94712	−26.51000	26.593	0.447	27.114	0.365	25.698	0.407
1	3	244.283	346.499	245.93696	−26.51695	26.160	0.532	26.993	0.263	25.135	0.136
1	3	432.803	360.296	245.94192	−26.51425	25.732	0.592	25.183	0.076	24.452	0.111
1	3	224.606	401.446	245.93745	−26.51850	26.183	0.516	26.922	0.261	25.795	0.395
1	3	40.860	558.464	245.93569	−26.52491	26.835	0.709	26.876	0.280	25.788	0.225
1	3	502.923	641.896	245.94870	−26.51943	26.616	0.616	26.546	0.226	24.972	0.216
1	3	436.170	672.737	245.94758	−26.52118	25.866	0.294	26.140	0.185	24.969	0.224
1	3	189.628	701.455	245.94194	−26.52573	26.929	0.829	26.690	0.239	25.559	0.366
1	3	505.508	713.059	245.95002	−26.52097	26.210	0.871	25.687	0.151	24.661	0.087
1	3	49.299	770.546	245.93968	−26.52944	26.500	0.901	26.461	0.205	25.420	0.181
1	3	387.762	772.659	245.94813	−26.52417	23.911	0.096	23.867	0.041	23.470	0.050
1	3	57.666	565.498	245.93623	−26.52480	25.037	0.277	24.529	0.059	23.907	0.072
1	3	480.416	632.276	245.94796	−26.51958	24.964	0.243	24.517	0.080	23.862	0.063
1	4	175.795	75.643	245.92793	−26.51705	23.453	0.159	23.479	0.033	23.010	0.072
1	4	786.603	89.207	245.93823	−26.53098	23.386	0.164	23.467	0.109	23.123	0.076
1	4	475.114	101.041	245.93255	−26.52417	24.035	0.383	24.343	0.142	23.868	0.250
1	4	98.263	353.537	245.91958	−26.51965	22.816	0.077	22.922	0.092	22.573	0.123
1	4	511.988	366.376	245.92652	−26.52917	22.726	0.091	22.824	0.048	22.619	0.050
1	4	69.449	406.796	245.91773	−26.51984	23.753	0.129	23.922	0.090	23.616	0.134
1	4	149.575	406.909	245.91914	−26.52164	22.199	0.110	22.672	0.048	22.521	0.098
1	4	551.871	468.205	245.92466	−26.53167	24.000	0.178	23.789	0.104	23.253	0.063
1	4	53.621	481.753	245.91558	−26.52067	22.703	0.143	23.148	0.360	22.786	0.190

Table 1—Continued

F	C	X	Y	RA	DEC	U	$\sigma(U)$	V	$\sigma(V)$	I	$\sigma(I)$
1	4	426.446	507.664	245.92146	−26.52947	22.282	0.032	22.482	0.065	22.302	0.130
1	4	582.420	509.741	245.92416	−26.53301	24.065	0.189	24.091	0.102	23.668	0.151
1	4	485.875	56.592	245.93385	−26.52372	26.562	9.999	26.145	0.219	25.122	0.210
1	4	431.788	73.207	245.93249	−26.52276	25.163	0.578	25.188	0.160	24.462	0.297
1	4	127.921	111.954	245.92617	−26.51655	25.322	0.565	25.133	0.207	24.216	0.133
1	4	425.910	187.107	245.92953	−26.52441	27.013	0.450	27.106	0.408	25.913	0.249
1	4	252.792	193.502	245.92633	−26.52061	24.919	0.356	24.606	0.114	24.147	0.150
1	4	597.301	192.798	245.93238	−26.52836	26.010	1.918	25.930	0.260	24.868	0.203
1	4	79.875	202.238	245.92306	−26.51688	24.626	0.223	24.618	0.230	23.816	0.182
1	4	479.737	202.983	245.93008	−26.52587	26.082	0.940	25.363	0.227	24.382	0.128
1	4	381.679	244.865	245.92730	−26.52432	26.655	0.566	27.196	0.400	25.786	0.350
1	4	675.266	245.385	245.93243	−26.53094	24.788	0.710	25.411	0.418	24.774	0.258
1	4	530.809	277.742	245.92909	−26.52820	25.697	0.122	25.675	0.125	24.650	0.132
1	4	562.166	557.753	245.92259	−26.53331	25.701	0.316	25.564	0.153	24.678	0.157
1	4	55.253	597.785	245.91270	−26.52253	25.825	0.259	25.657	0.179	24.663	0.336
1	4	641.403	650.327	245.92166	−26.53652	24.299	0.279	24.029	0.088	23.545	0.099
1	4	733.349	491.053	245.92727	−26.53610	22.652	0.099	22.971	0.071	22.688	0.067
6	1	109.020	695.087	245.97895	−26.53500	28.649	0.235	26.935	0.239
6	1	617.842	763.431	245.97259	−26.53798	26.587	0.130	25.246	0.150
6	1	647.847	734.535	245.97248	−26.53849	27.841	0.296	26.118	0.418
6	1	792.862	664.476	245.97142	−26.54026	24.265	0.054	23.875	0.130
6	1	786.000	330.415	245.97416	−26.54362	27.162	0.136	25.869	0.394
6	1	119.092	548.364	245.98002	−26.53656	27.588	0.263	26.132	0.415
6	1	151.445	436.138	245.98057	−26.53794	27.686	0.349	26.615	0.464
6	1	400.669	187.882	245.97973	−26.54229	28.201	0.423	25.858	0.341

Table 1—Continued

F	C	X	Y	RA	DEC	U	$\sigma(U)$	V	$\sigma(V)$	I	$\sigma(I)$
6	1	446.760	587.896	245.97595	−26.53853	28.963	0.431	26.778	0.370
6	1	89.280	119.655	245.98382	−26.54073	28.055	0.486	26.965	0.433
6	2	213.671	440.346	245.99232	−26.53079	24.794	0.055	24.174	0.056
6	2	501.645	226.109	245.98181	−26.52782	27.130	0.215	25.448	0.220
6	2	364.132	350.366	245.98738	−26.52889	25.013	0.071	24.490	0.100
6	2	218.394	353.329	245.99006	−26.53209	27.510	0.303	26.000	0.308
6	2	296.210	424.047	245.99043	−26.52922	26.407	0.164	25.202	0.303
6	2	155.546	432.165	245.99316	−26.53222	27.713	0.272	26.129	0.417
6	2	687.260	450.714	245.98408	−26.52008	27.791	0.316	25.924	0.417
6	2	616.300	163.544	245.97821	−26.52627	26.144	0.136	25.460	0.302
6	2	669.101	487.989	245.98533	−26.51988	25.999	0.118	24.970	0.256
6	2	339.665	656.623	245.99543	−26.52451	23.852	0.054	23.382	0.033
6	2	310.651	30.680	245.98039	−26.53520	25.046	0.089	24.455	0.135
6	2	743.250	713.292	245.98959	−26.51468	25.020	0.070	24.317	0.067
6	2	688.943	207.703	245.97801	−26.52394	23.563	0.050	23.284	0.063
6	2	584.775	261.926	245.98121	−26.52539	27.762	0.484	26.105	0.233
6	2	346.208	322.712	245.98701	−26.52973	25.254	0.096	24.414	0.124
6	2	115.407	363.480	245.99217	−26.53421	24.599	0.044	24.007	0.109
6	2	372.111	269.815	245.98522	−26.53001	27.146	0.230	25.552	0.256
6	2	55.610	392.199	245.99395	−26.53508	25.887	0.121	24.823	0.116
6	2	80.709	466.106	245.99534	−26.53333	24.491	0.060	23.826	0.089
6	2	428.026	473.008	245.98928	−26.52549	27.819	0.356	26.035	0.328
6	2	581.605	514.089	245.98754	−26.52141	24.985	0.084	24.003	0.054
6	2	443.064	756.434	245.99604	−26.52062	26.491	0.104	25.396	0.287
6	2	545.114	157.697	245.97933	−26.52795	26.066	0.152	24.969	0.117

Table 1—Continued

F	C	X	Y	RA	DEC	U	$\sigma(U)$	V	$\sigma(V)$	I	$\sigma(I)$
6	2	381.549	598.594	245.99325	−26.52450	25.399	0.119	24.375	0.072
6	2	766.326	182.357	245.97602	−26.52264	24.189	0.060	23.698	0.144
6	2	187.376	315.015	245.98966	−26.53340	26.668	0.185	25.688	0.183
6	2	771.348	629.224	245.98701	−26.51539	27.322	0.320	26.049	0.450
6	2	645.273	316.654	245.98149	−26.52316	25.616	0.118	24.984	0.202
6	2	150.198	792.555	246.00216	−26.52654	26.335	0.249	24.931	0.255
6	2	770.776	492.578	245.98362	−26.51757	27.718	0.397	25.552	0.238
6	2	772.211	143.847	245.97497	−26.52313	27.706	0.391	26.191	0.361
6	2	147.354	37.054	245.98349	−26.53871	28.302	0.444	26.948	0.252
6	2	506.589	606.140	245.99119	−26.52160	28.724	0.386	26.400	0.277
6	3	262.622	593.039	246.00041	−26.54964	24.766	0.075	24.147	0.111
6	3	630.487	217.970	246.00292	−26.53540	23.322	0.034	23.059	0.054
6	3	222.354	333.591	245.99478	−26.54449	25.286	0.066	24.417	0.097
6	3	195.767	340.510	245.99423	−26.54506	26.895	0.208	25.778	0.328
6	3	667.068	417.219	246.00738	−26.53927	22.875	0.028	22.611	0.058
6	3	408.539	463.328	246.00175	−26.54442	26.501	0.171	25.556	0.324
6	3	627.251	541.452	246.00860	−26.54268	27.666	0.482	26.150	0.374
6	3	423.940	550.251	246.00369	−26.54612	26.892	0.158	25.734	0.190
6	3	101.633	581.690	245.99620	−26.55194	28.187	0.379	27.142	0.022
6	3	187.459	189.255	245.99134	−26.54181	25.392	0.087	24.694	0.068
6	3	393.543	257.209	245.99769	−26.54004	27.713	0.171	26.708	0.451
6	3	332.736	348.005	245.99780	−26.54305	25.672	0.112	24.319	0.081
6	3	606.532	225.956	246.00247	−26.53596	25.291	0.090	24.420	0.128
6	3	198.302	611.158	245.99913	−26.55107	26.940	0.198	25.518	0.193
6	3	418.869	54.389	245.99473	−26.53512	26.916	0.159	25.636	0.325

Table 1—Continued

F	C	X	Y	RA	DEC	U	$\sigma(U)$	V	$\sigma(V)$	I	$\sigma(I)$
6	3	744.329	60.791	246.00296	−26.53014	27.630	0.461	26.216	0.478
6	3	727.209	605.294	246.01220	−26.54251	27.447	0.359	26.069	0.293
6	3	260.083	756.515	246.00326	−26.55331	28.444	0.471	25.997	0.381
6	3	443.506	256.110	245.99893	−26.53922	27.988	0.239	27.084	0.431
6	3	512.311	785.385	246.01005	−26.54994	24.011	0.046	23.562	0.084
6	3	329.639	80.680	245.99296	−26.53713	26.773	0.173	25.254	0.164
6	3	372.806	726.795	246.00555	−26.55086	28.247	0.465	26.754	0.319
6	3	333.206	556.964	246.00154	−26.54772	28.243	0.463	27.226	0.351
6	3	525.824	61.948	245.99754	−26.53359	28.779	0.398	27.736	0.040
6	4	384.522	396.847	245.98210	−26.55425	25.471	0.080	24.456	0.094
6	4	292.027	470.782	245.97862	−26.55332	24.063	0.048	23.594	0.091
6	4	74.221	478.114	245.97462	−26.54854	24.763	0.074	24.132	0.062
6	4	267.537	610.828	245.97467	−26.55498	25.634	0.098	24.820	0.134
6	4	375.318	651.784	245.97552	−26.55805	24.606	0.073	24.003	0.075
6	4	318.687	689.369	245.97358	−26.55736	27.140	0.187	25.708	0.247
6	4	516.217	349.434	245.98562	−26.55647	26.465	0.184	25.199	0.363
6	4	433.489	254.303	245.98657	−26.55311	27.964	0.391	26.115	0.250
6	4	227.322	551.512	245.97544	−26.55314	24.085	0.059	23.564	0.047
6	4	474.078	757.921	245.97460	−26.56192	25.987	0.118	25.229	0.173
6	4	503.497	709.015	245.97634	−26.56182	26.555	0.153	25.545	0.192
6	4	321.260	795.249	245.97100	−26.55907	23.881	0.049	23.449	0.101
6	4	198.675	547.455	245.97505	−26.55243	27.103	0.274	25.268	0.173
6	4	54.743	669.624	245.96949	−26.55112	25.083	0.087	24.219	0.149
6	4	737.255	474.875	245.98632	−26.56340	24.188	0.045	23.629	0.048
6	4	75.411	141.795	245.98309	−26.54331	27.613	0.315	25.508	0.399

Table 1—Continued

F	C	X	Y	RA	DEC	U	$\sigma(U)$	V	$\sigma(V)$	I	$\sigma(I)$
6	4	733.469	284.282	245.99105	−26.56033	25.511	0.128	24.725	0.175
6	4	573.187	739.180	245.97682	−26.56385	25.428	0.092	24.507	0.214
6	4	50.351	191.311	245.98141	−26.54352	24.129	0.058	23.698	0.095
6	4	80.333	140.315	245.98321	−26.54340	27.351	0.274	25.979	0.386
6	4	184.874	64.031	245.98696	−26.54455	26.729	0.134	25.706	0.386
6	4	715.109	793.780	245.97795	−26.56784	25.770	0.115	24.810	0.147
6	4	120.982	766.116	245.96824	−26.55412	28.571	0.172	27.472	0.375
6	4	256.210	277.789	245.98284	−26.54948	27.950	0.417	26.691	0.434